NASA Technical Paper 3652

On Determining the Rise, Size, and Duration Classes of a Sunspot Cycle

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TECHNICAL PAPER

ON DETERMINING THE RISE, SIZE, AND DURATION CLASSES OF A SUNSPOT CYCLE

I. INTRODUCTION

About 60 years ago, Waldmeier[1] showed that the shape of the sunspot cycle curve for a given cycle is primarily determined by the height of its maximum (cf. Kiepenheuer[2]). In particular, he found that larger amplitude cycles reached maximum more quickly than smaller amplitude cycles. Thus, there appears to be an inverse relationship between the size of the cycle and its ascent duration. Today, we recognize this relationship as the Waldmeier effect, so-named by Bracewell[3] (cf. Wilson[4]).

In the same paper, Waldmeier[1] also noted that even-numbered and odd-numbered sunspot cycles appeared to behave differently. In particular, he found that even-numbered cycles tended to be of smaller amplitude and longer ascent duration than odd-numbered cycles.

More recently, Rabin et al.[5] and Wilson[6] found that cycle duration or period seems to be distributed bimodally; i.e., sunspot cycles are better described in terms of being either of short period (period <11 years) or of long period (period >11 years). Also, Wilson[4] found that a marginally significant correlation exists between period and ascent duration, one that relates shorter than average ascent duration (fast rising) with shorter than average cycle length (short period) and longer than average ascent duration (slow rising) with longer than average cycle length (long period). In particular, he found that when the cycle was a fast-rising cycle and even-numbered, it usually was of short period (3 of 4), and when the cycle was a slow-rising cycle and even-numbered, it usually was of long period (5 of 6). Similarly, when the cycle was a fast-rising cycle and odd-numbered, it usually was of short period (4 of 6), and when the cycle was a slow-rising cycle and odd-numbered, it usually was of long period (3 of 5).

In this paper, we investigate the interrelationships of ascent duration, maximum amplitude, period, and cycle numberedness (even-odd) for cycles 1 to 21. Based on the inferred correlations and the early behavior of cycle 22, we characterize cycle 22 in terms of its probable rise, size, and duration classes. We find that the early behavior of cycle 22 suggested that it probably would be a fast-rising, large-amplitude, short-period cycle. Two of these parameter classes (rise and maximum amplitude) have already been borne out, while the third awaits the occurrence of conventional onset for cycle 23 (i.e., the minimum value of smoothed sunspot number prior to the rise to maximum), which is expected before April 1997 if, indeed, cycle 22 is a short-period cycle. Application of this method to cycle 23 should allow a an early determination of its rise, maximum amplitude, and period classes probably by early-to-mid 1998, presuming, of course, an onset of cycle 23 before April 1997 (Wilson et al.[7 8]).

II. RESULTS

Figure 1 depicts the variations of maximum amplitude RM (based on smoothed sunspot number), ascent duration ASC (in months), and cycle duration or period PER (in months) as a function of sunspot cycle number SCN for cycles 1 to 21. The values were extracted from a tabular listing of sunspot cycle parameters in Wilson et al.[8] The median values of RM (= 110.6) and ASC (= 48 months) are identified. For RM when a value is equal to or more than the median value, we denote it as a large-amplitude cycle; when the value is less than the median value, we call it a small-amplitude cycle. Similarly, for ASC when a value is equal to or more than the median value, we denote it as a slow-rising cycle; when the value is less than the median, we call it a fast-rising cycle. For PER, we note that it appears to be separable into two characteristic subgroupings: a short-period cycle group (PER <127 months) and a long-period cycle group (PER >134 months), with an 8-month gap that spans the entire record (cf. Rabin et al.[5]; Wilson[6]).

Figure 2 combines the three parameters in the form of a 2 by 2 contingency table (based on the median values of RM and ASC), where the numbers identified in the individual quadrants refer to the SCN of each cycle and the circled numbers signify those cycles that are of long period. Ignoring bimodal class and cycle numberedness, we see that sunspot cycles neatly separate into two predominant rise-amplitude classes: fast-large cycles (quadrant IV) and slow-small cycles (quadrant II). This is the Waldmeier effect. For cycles 1 to 21, 18 of 21 cycles strictly obey the inferred inverse correlation, associating fast (slow) rise time with large (small) maximum amplitude. Listed below the ASC-RM diagram are the various 2 by 2 contingency tables involving ASC-RM, ASC-PER, and ASC-SCN, as well as, two three-dimensional (3-D) contingency tables involving ASC-RM-PER and ASC-RM-SCN.

We can easily evaluate the statistical significance of the 2 by 2 contingency tables, shown in figure 2, by means of the Fisher's exact test (Everitt;[9] p. 15). In figure 3, we show the results of such statistical testing. For ASC-RM, we note that its distribution (quadrants I-II-III-IV) is 2:9:1:9; thus, the probability of obtaining the observed table (ignoring bimodal class and cycle numberedness), or one more suggestive of a departure from independence (chance), is computed to be P = 0.2 percent. Thus, when a cycle is viewed as being a fast riser, it probably will be of large amplitude (9 of 10), and when a cycle is viewed as being a slow riser, it probably will be of small amplitude (9 of 11). For ASC-PER, we compute P = 6.3 percent, a marginally significant result, and for ASC-SCN, we compute P = 41.0 percent. In figure 3, we also show the 2 by 2 contingency tables of RM-PER, RM-SCN, and PER-SCN; none of these distributions appears to be statistically important.

Figure 4 displays the 3-D contingency table for ASC-RM-PER, where the rows are ASC classes (fast/slow), the columns are RM classes (large/small), and the layers are PER classes (short/long). An evaluation of the statistical significance of the 3-D contingency table can easily be accomplished by means of the chi-square test statistic (Everitt;[9] p. 71). The result of such testing is tabulated below the contingency table. The chi-square statistic is computed to be 18.59 that, when compared to the chi-square test statistic (= 9.49 for 4 degrees-of-freedom at the 5-percent level of significance or 95-percent level of confidence), is found to be significantly larger than the test statistic and, by means of hypothesis testing, strongly suggests that the null hypothesis of mutual independence must be rejected. Thus, it appears statistically important that fast-large (slow-small) cycles are more likely to be of short (long) period. Based on the 3-D table, we see that when a cycle is portrayed to be a fast-large cycle, it is more than three times likely that it will also be a short-period cycle; when a cycle is portrayed to be a slow-small cycle, it is twice as likely to be a long-period cycle.

A similar analysis is shown in figure 5 for ASC-RM-SCN, where the layering is now the SCN (i.e., even-odd numberedness). Again, we find that the null hypothesis must be rejected. The distribution appears statistically important. An even-numbered cycle is more likely to be of small amplitude when it is portrayed as a slow riser (5 of 6) and more likely to be of large amplitude when it is portrayed as a fast riser (4 of 4). Similarly, an odd-numbered cycle is more likely to be of small amplitude when it is portrayed as a slow riser (4 of 5) and more likely to be of large amplitude when it is portrayed as a fast riser (5 of 6).

III. DISCUSSION AND CONCLUSIONS

Section II has demonstrated that the ascent durations, maximum amplitudes, periods, and numberedness of sunspot cycles 1 to 21 are not mutually independent. The size of a sunspot cycle is clearly linked to the quickness of its rise (the Waldmeier effect). Also, the bimodal class of a sunspot cycle appears closely related to its rise time class, with short-period cycles being more closely associated with fast-rising, large-amplitude cycles and long-period cycles being more closely associated with slow-rising, small-amplitude cycles. Thus, if we could know early in the cycle whether or not it could be characterized as being fast or slow rising, then we could also have an indication of the expected size (amplitude class) of the cycle and its length (bimodal class).

Figure 6 shows the behavior of fast- and slow-rising cycles in comparison to the mean curve for cycles 1 to 21 by means of histograms for the first 12 months of a sunspot cycle following onset. Fast cycles are denoted by the cross-hatched pattern. Plotted are the number of fast- and slow-rising cycles that had a smoothed sunspot number for elapsed time t from cycle onset either above (denoted a) or below (denoted b) the mean curve (based on an epoch analysis approach, using minimum amplitude Rm occurrence as the origin). At $t = 0$ (i.e., Rm occurrence), 6 of 10 fast-rising cycles had their $R(0) \geq 5.7$; similarly, 8 of 11 slow-rising cycles had their $R(0) < 5.7$. Thus, based strictly on a cycle's Rm value, we have about a two-thirds chance of guessing correctly whether a cycle is a fast riser or a slow riser. As the cycle progresses, more of the fast-rising cycles have their respective $R(t)$ values greater than or equal to $<R(t)>$ and more of the slow-rising cycles have their respective $R(t)$ values less than $<R(t)>$. By the 12th month of the cycle, 9 of 10 fast-rising cycles have their $R(12) \geq <R(12)>$ and 10 of 11 slow-rising cycles have their $R(12) <<R(12)>$. So, it appears that by the 12th month of a sunspot cycle, we have a fairly strong idea whether or not a cycle is going to be a fast riser or a slow riser. Consequently, we also have a fairly strong idea whether a cycle is going to be of large or small amplitude and of short or long period, respectively.

Figure 7 depicts the behavior (bottom panel) of cycle 22 from its onset (September 1986) through July 1995 and of the mean curve based on cycles 1 to 21. It should be noted that cycle 22 had the highest Rm of any cycle on record. Consequently, at $t = 0$, $R(0) ><R(0)>$. $R(t)$ continued throughout the rise phase to be greater than $<R(t)>$. So, at $t = 12$, $R(12) ><R(12)>$ and we should have recognized that we were on sound statistical footing, that cycle 22 was very probably going to be a fast-rising cycle that, indeed, turned out to be true (ASC = 34 months, the fastest rise time on record). Because cycle 22, was very probably a fast-rising cycle, we also should have recognized that it would be a large-amplitude cycle that, likewise, turned out to be true (RM = 158.5, tied for third in terms of size).

We now know that cycle 22 is described as an even-numbered, fast-rising, large-amplitude cycle. Statistically speaking, we should then also expect it to be a short-period cycle, having PER <127 months. Wilson et al.[7] have shown that the decline of cycle 22 is consistent with the notion that it is a short-period cycle, having a period of about 123±3 months, suggesting an onset for cycle 23 of about December 1996 (±3 months). Supporting this is the finding of Wilson,[10] based on the inferred slope for the declining portion of cycle 22 that was gleaned from its observed slope during the rising portion, and of Wilson,[11] based on the occurrence of the first spotless day during the decline of the cycle.

The top portion of figure 7 compares cycle 22 against the mean of cycles 1 to 21 in two ways: (1) as a deviation in terms of standard deviation units and (2) as a ratio (observed/mean). In terms of standard deviation units, the early behavior of cycle 22 suggested that its smoothed sunspot numbers were typically outside the range of observed values that had been seen for cycles 1 to 21 and that cycle 22 was a >+2 standard deviation cycle. As it turned out, cycle 22 was a +1.6 standard deviation cycle (based on the mean curve). Cycle 22 remained above the mean curve until about June 1993. It has continued to track below the mean curve into mid 1995. Presuming cycle 22 to be a short-period cycle, the deviation should begin to return to values closer to zero, if not above zero, very soon.

In terms of the ratio, it is remarkable that there appears to have been long intervals of time when the ratio was essentially constant. For example, for nearly the first 3 years of the cycle, the ratio averaged about 2. By offsetting the mean curve by this factor, we could easily have predicted the actual shape of the cycle during its rise. Another long interval of near constant ratio occurred between January 1990 and January 1992, when the ratio hovered about 1.5. Present values are running about 0.7 to 0.8 of the mean. We note that to the eye, instead of viewing, the ratio has been composed of long intervals of near constant value punctuated every 2 to 3 years by brief episodes lasting a few to several months when the bulk of the change occurs, we could also describe the ratio in terms of a smoothly declining slope of about –0.15 units every 10 months or so. Together, these findings suggest the fascinating possibility that we might be able to predict the shape of the cycle by means of a template design (in particular, the mean curve), utilizing an appropriate offset. In Hathaway et al.,[12] we demonstrate that one can predict the general shape of the sunspot cycle, including the interval encompassing the maximum phase and the several year descending portion, fairly well from about 2 years into the cycle.

Looking to cycle 23, to properly assess its rise, amplitude, and period classes, we must adjust the data set to include the results from cycle 22. Therefore, for the Waldmeier effect (ASC-RM), we note that the median RM value shifts slightly upward from 110.6 to 113.2. The median ASC value remains the same (= 48 months). The original distribution of 2:9:1:9 (fig. 2) becomes 1:10:1:10. The slightly higher median of RM causes cycle 20 to move from quadrant I to quadrant II and cycle 22 to fall within quadrant IV. The probability of obtaining this distribution, or one more suggestive of a departure from independence, is $P = 0.02$ percent. Thus, if cycle 23 turns out to be a fast or slow riser, then we very likely expect it to be of large or small amplitude, respectively (where large amplitude means ≥113.2).

Concerning the period of cycle 23, presuming that cycle 22 turns out to be a short-period cycle, the original distributions of 2:6:1:2 (long period) and 0:3:0:7 (short period) (fig. 2) become 1:7:1:2 (long period) and 0:3:0:8 (short period), and the original distribution for ASC-PER of 8:3:7:3 becomes 8:3:8:3, yielding $P = 4.3$ percent, a statistically significant result. Thus, if cycle 23 turns out to be a fast or slow riser, then we also expect it to be of short or long period, respectively.

The decision rule to regard cycle 23 as a fast or slow riser, based upon its $R(12)$ value in comparison to $\langle R(12)\rangle$ (fig. 6) is slightly softer by including cycle 22. The $\langle R(12)\rangle$ value changes from 18.0 to 19.0, and the fraction of fast risers changes from 9 of 10 to 9 of 11 (when $R(12) \geq \langle R(12)\rangle$). Waiting until the 16th month, we again have the condition that all but one of the cycles fit the inferred paradigm of being a fast riser when $R(\mathrm{t}) \geq \langle R(t)\rangle$ and being a slow riser when $R(t) < \langle R(t)\rangle$. The mean value for cycles 1 to 22 of $t = 16$ is $\langle R(16)\rangle = 28.9$.

In conclusion, this study has shown that ascent duration, maximum amplitude, and period are not mutually independent. Instead, both maximum amplitude and period appear to be related to ascent duration, but in the opposite sense. Maximum amplitude is inversely related to ascent duration (the Waldmeier effect), suggesting that fast-rising cycles (ASC less than its median value) almost always are large-amplitude cycles (RM equal to or larger than its median value) and that slow-rising cycles almost always are small-amplitude cycles. For cycles 1 to 22, 20 of 22 cycles fit this paradigm. On the other hand, period is directly related to ascent duration, suggesting that fast-rising cycles usually are short-period cycles (PER <127 months) and that slow-rising cycles usually are long-period cycles (PER >134 months). For cycles 1 to 21, 15 of 21 cycles fit this paradigm (if cycle 22 turns out to be a short-period cycle, then 16 of 22 are found to conform to the inferred pattern). We also found that the character of the cycle (fast riser or slow riser) can be determined early in the cycle, usually by the 12th month following onset, deduced by a comparison of smoothed sunspot number to the mean curve: A fast-rising cycle usually has a smoothed sunspot number greater than that of the mean curve, while a slow-rising cycle usually has a smoothed sunspot number less than the mean curve. The behavior of cycle 22 is such that we believe it to be a short-period cycle (we already know that cycle 22 is an even-numbered, fast-rising, large-amplitude cycle, indicative of short-period cycles); hence, onset for cycle 23 should occur before April 1997. The behavior of cycle 22 also suggests that a template method for predicting the size and shape of the sunspot cycle may not be unreasonable.

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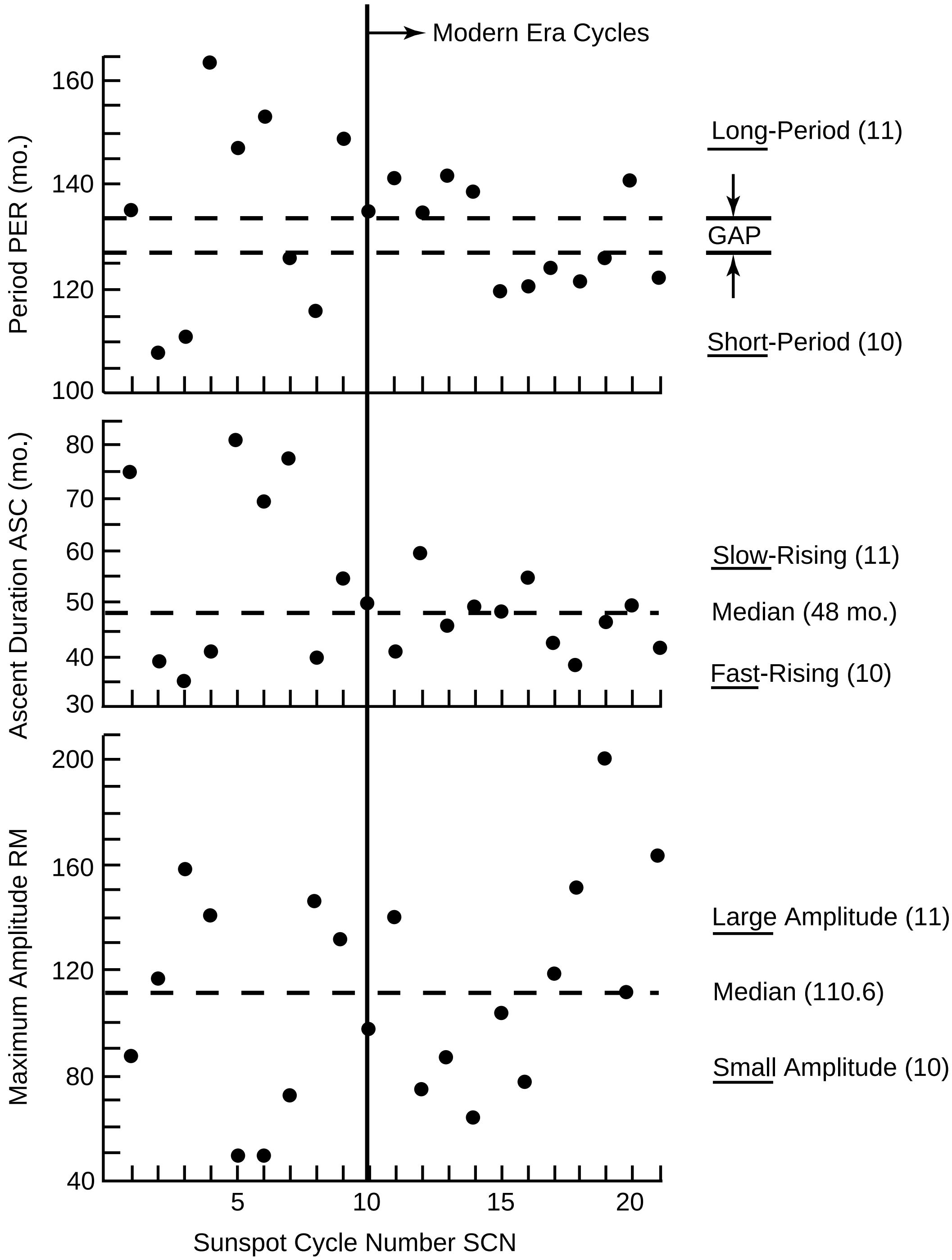
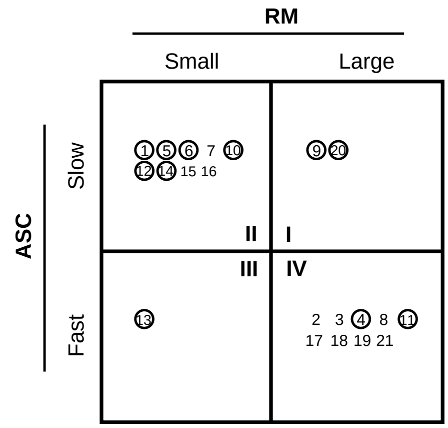


Figure 1. The variation of maximum amplitude (RM), ascent duration (ASC), and period (PER) for cycles 1 to 21. RM is expressed in smoothed sunspot number. ASC and PER are expressed in months. The median values for RM and ASC are shown, subdividing each into two regimes: above the median (slow rising and large amplitude) and below the median (fast rising and small amplitude). PER appears separable into short-period (PER <127 months) and long-period (PER >134 months) cycles. The overall sunspot record can be divided into a pre- and post-modern era, based on the completeness of the sunspot daily record. Cycles beginning with cycle 10 are referred to as modern era cycles.



Grouping	RM Small	RM Large	PER Short	PER Long	SCN Even	SCN Odd
Slow	9	2	3	8	6	5
Fast	1	9	7	3	4	6
Slow–Small			3	6	5	4
Slow–Large			0	2	1	1
Fast–Small			0	1	0	1
Fast–Large			7	2	4	5

Figure 2. The ASC-RM contingency table, including bimodality and SCN. The numbers in the contingency table refer to the SCN. Circled numbers refer to cycles that are of long period.

Testing for Mutual Independence Involving Two Variables

Comparsion Group	Distribution	Fisher's Exact Test Probability
ASC–RM	2:9:1:9	0.2%
ASC–PER	8:3:7:3	6.3%
ASC–SCN	5:6:4:6	41.0%
RM–PER	4:7:3:7	13.5%
RM–SCN	6:5:5:5	59.0%
PER–SCN	5:6:4:6	41.0%

Figure 3. Testing for mutual independence involving two variables (the Fisher's exact test) for selected groupings of cycles. The distribution refers to the distribution by quadrant (I-II-III-IV), constructed similarly to that shown in figure 2. The distributions are determined from parametric median values. The results of Fisher's exact test for 2 by 2 contingency tables are displayed. When $P \leq 5$ percent, the result is statistically significant; when $P \leq 10$ percent, it is marginally significant.

Testing for Mutual Independence Between ASC, RM, and PER

		RM ≥ 110.6				
		No (Small)		**Yes (Large)**		
	Period Class	Short	Long	Short	Long	**Totals**
ASC ≥ 48 mo.	Yes (Slow)	3	6	0	2	**11**
	No (Fast)	0	1	7	2	**10**
Totals		**3**	**7**	**7**	**4**	**21**

Table Position	**Grouping**	**fa**	**fe**	**D**	**DXD**	**DXD/fe**
(1,1)	Slow-Small-Short	3	2.49	0.51	0.26	0.10
(1,2)	Slow-Small-Long	6	2.74	3.26	10.63	3.88
(1,3)	Slow-Large-Short	0	2.74	–2.74	7.51	2.74
(1,4)	Slow-Large-Long	2	3.02	–1.02	1.04	0.34
(2,1)	Fast-Small-Short	0	2.27	–2.27	5.15	2.27
(2,2)	Fast-Small-Long	1	2.49	–1.49	2.22	0.89
(2,3)	Fast-Large-Short	7	2.49	4.51	20.34	8.17
(2,4)	Fast-Large-Long	2	2.74	–0.74	0.55	0.20
	Totals	**21**				$18.59 = \chi^2$

df = 4

$\chi^2_{0.05} = 9.49$ $\chi^2 > \chi^2_{0.05} \Rightarrow$ Reject Null Hypothesis

Figure 4. Testing for mutual independence between ASC, RM, and PER. In the analysis of the 3–D contingency table, fa refers to the actual frequency of occurrence, fe to the expected frequency of occurrence (based on marginal totals), and D to the difference (fa – fe). The sum of DXD/fe is the chi-square parameter. The term “df” refers to the degrees of freedom, here equal to 4, with the chi-square test statistic equal to 9.49 (for the 95-percent level of confidence or 5-percent level of significance). Because the observed chi-square parameter exceeds the chi-square test statistic, by hypothesis testing, we reject the null hypothesis that the distribution is due entirely to chance.

Testing for Mutual Independence Between ASC, RM, and SCN

		RM ≥ 110.6				
		No (Small)		**Yes (Large)**		
	SCN Class	Even	Odd	Even	Odd	**Totals**
ASC ≥ 48 mo.	Yes (Slow)	5	4	1	1	**11**
	No (Fast)	0	1	4	5	**10**
	Totals	**5**	**5**	**5**	**6**	**21**

Table Position	**Grouping**	**fa**	**fe**	**D**	**DXD**	**DXD/fe**
(1,1)	Slow-Small-Even	5	2.49	2.51	6.30	2.53
(1,2)	Slow-Small-Odd	4	2.74	1.26	1.59	0.58
(1,3)	Slow-Large-Even	1	2.74	–1.74	3.03	1.10
(1,4)	Slow-Large-Odd	1	3.02	–2.02	4.08	1.35
(2,1)	Fast-Small-Even	0	2.27	–2.27	5.15	2.27
(2,2)	Fast-Small-Odd	1	2.49	–1.49	2.22	0.89
(2,3)	Fast-Large-Even	4	2.49	1.51	2.28	0.92
(2,4)	Fast-Large-Odd	5	2.74	2.26	5.11	1.86
	Totals	**21**				$11.51 = \chi^2$

df = 4

$\chi^2_{0.05} = 9.49$ $\quad \chi^2 > \chi^2_{0.05} \Rightarrow$ Reject Null Hypothesis

Figure 5. Testing for mutual independence between ASC, RM, and SCN. The construction and interpretation follow that shown in figure 4.

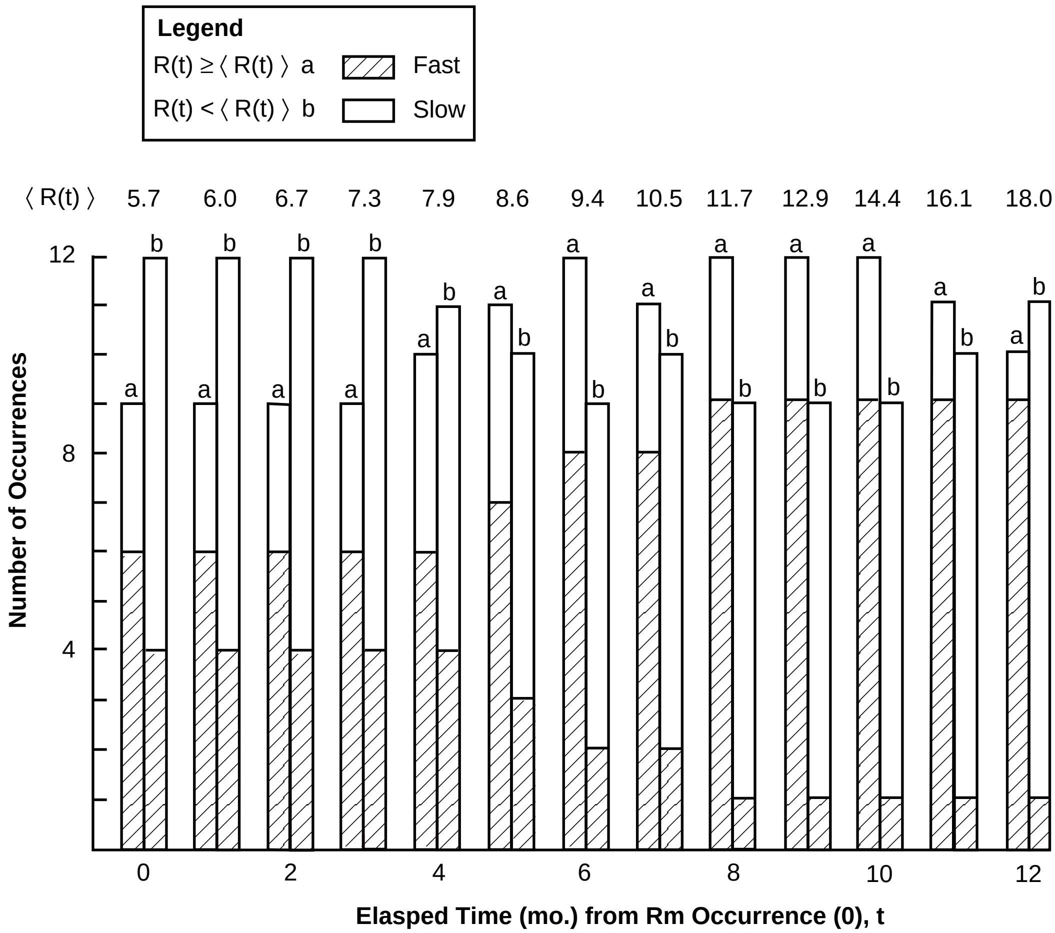


Figure 6. Histogram of fast-rising and slow-rising cycles as a function of elapsed time in months from cycle onset (minimum amplitude Rm occurrence), based on a comparison of smoothed sunspot number for time t, denoted $R(t)$, with the mean curve (cycles 1 to 21), denoted $<R(t)>$.

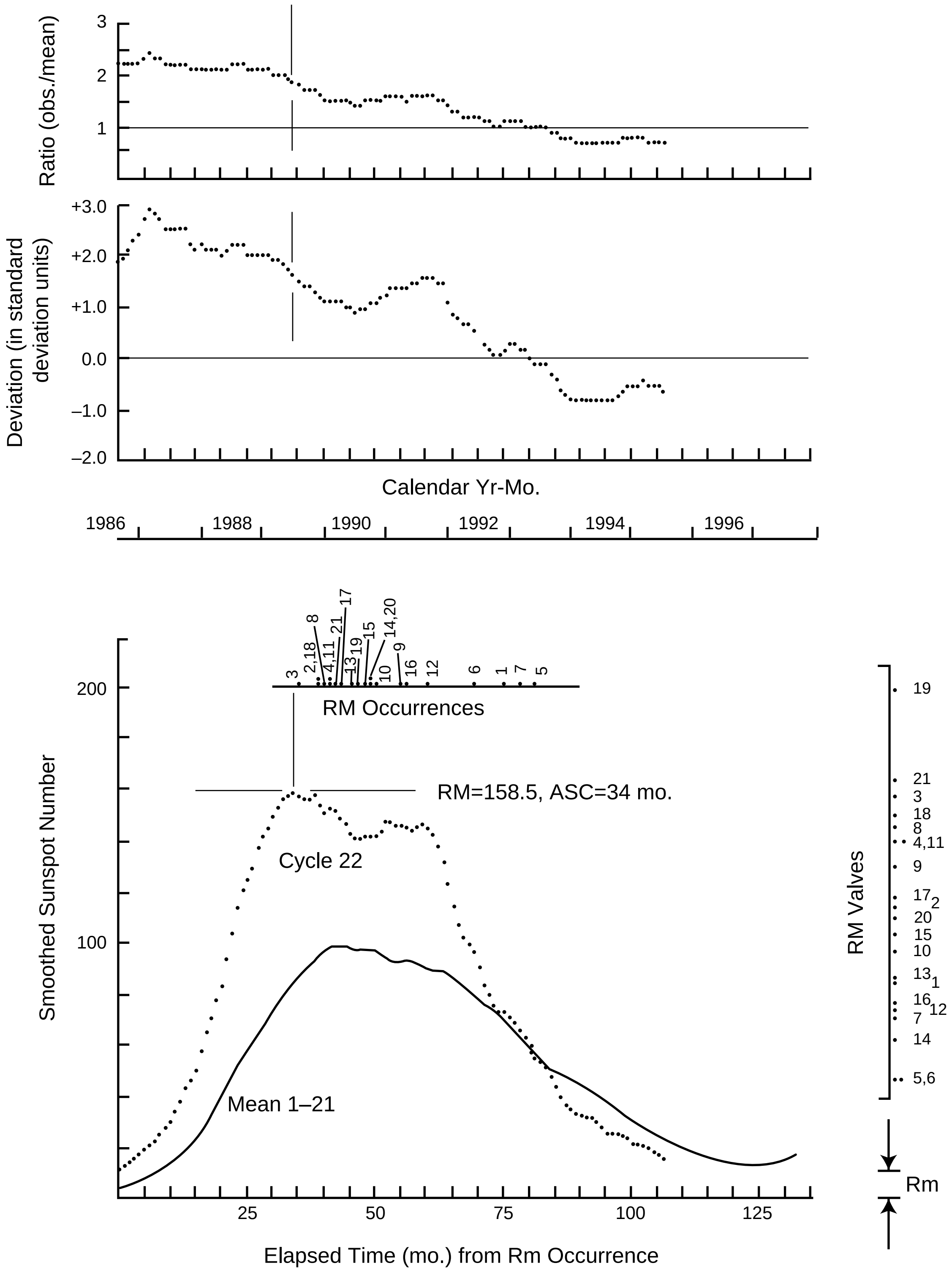


Figure 7. Comparison of smoothed sunspot number values for cycle 22 with the mean curve (cycles 1 to 21). Bottom: The relative ascent durations and maximum amplitudes for cycles 1 to 21 are shown, as well as, the RM and ASC values for cycle 22. Top: The deviation of cycle 22 from the mean curve as a function of elapsed time in months from Rm occurrence, plotted in standard deviation units and as a ratio (observed value/mean curve value for same time). The vertical lines denote when cycle 22 reached peak.